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# Review of OKGs method for thermal power uncertainty determination using PROBERA

Institute for Energy Technology

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**Summary**

A review of the method used by OKG to determine thermal power and its uncertainty has been performed by IFE (Institutt for energiteknikk). This review starts with a background history describing IFE's activities and experience in parameter uncertainty determination.

The method used in PROBERA is compared with the standard VD1-2048. In addition some simple test cases are run.

Finally, the actual models for the O1, O2 and O3 reactors were considered. These were found to be correctly implemented. The cross correlations were investigated and found to be consistent with expectations. The relative contributions were also considered and the major contributory factors checked by direct calculation.

We therefore can say that the derived uncertainties of the thermal power are a reasonable representation of the true uncertainty.



**Keywords:** Data Reconciliation, VDI-2048, Thermal Power, Uncertainty determination


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|---|---|---|---|
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## Contents

# 1 Scope of this report.

A review of the method used by OKG to determine thermal power and its uncertainty has been performed by IFE (Institutt for energiteknikk). This review starts with a background history describing IFE's activities and experience in parameter uncertainty determination. One of the most important standards in this field, VDI-2048, is also described.

The actual implementation used at OKG, called PROBERA, is briefly described. OKG's own documentation provides a complete description of the system. This system is then compared to the standard procedures used. Method differences and special considerations for the models specific to OKG are discussed.

Then some comparison tests are defined and performed. These tests are designed to qualify the calculation method.

Finally, the actual implementations for the O1, O2 and O3 reactors are considered.

In order to perform this review a copy of the PROBERA system was provided for IFE together with the relevant parts of the source code. Documentation for the methods used and the calculation reports were also provided for inspection.

# 2 Background history

The method employed by OKG for thermal power uncertainty determination is often referred to as data-reconciliation. This method covers applications where process simulations (models) are used together with many process measurements in order to gain more information about the process and to provide a limited validation of the measurement values.

In this section we provide a general description of the data-reconciliation technique. In addition a summary of our (IFE) own experience with data-reconciliation as well as a description of an applicable standard is provided.

## 2.1 Data reconciliation (DR)

The technique used for comparing measurement values with the help of process information is often termed data reconciliation. The concept is easiest to understand when considering a simple example such as several measurements at the same point in the process. Here the process information is that the instruments are said to all be measuring the same quantity: process state. This process state can be determined by taking the average of all the measurements. The process state is then called the reconciled value of the measurements. The difference between measured value and reconciled value is termed the residual.

Given the uncertainties in the individual measurements an uncertainty in the average value can be easily determined. This is then called the reconciled uncertainty. In the example above there was what is termed physical redundancy. I.e. the process state could still be determined even if one measurement value was missing.

In general there are several measurements of the different physical properties at different physical locations. A process state is then defined for each physical property and location. This process information is then linked based on a physical description of the plant.

Sometimes this can allow the process state at one point to be completely determined at another point. This means that the measurement value at that point is not necessary, i.e. redundant. This example is termed analytical redundancy. A consequence of this redundancy is a reduction in the uncertainty of any process state whose determination is reliant on these measurements.

The improvement in the reconciled measurement uncertainties compared to the measurement uncertainties is termed the adjustability and gives the degree of redundancy.

## 2.2 Institutt for energiteknikk and the Halden Reactor Project

IFE is an international research institute for energy and nuclear technology. IFE's mandate is to undertake research and development, on an ideal basis and for the benefit of society, within the Energy and Petroleum sector and to carry out assignments in the field of nuclear technology for the nation.

IFE's nuclear technology comprises all activities that are directly or indirectly related to the Institute's two research reactors, in Halden and at Kjeller. The Institute for Energy Technology was founded in 1948 and is now an independent foundation. IFE is the host of the Halden Reactor Project.

The OECD-Halden Reactor Project is a joint undertaking of national organizations in 18 countries sponsoring a jointly financed programme under the auspices of the OECD - Nuclear Energy Agency. The programme's aim is to generate key information for safety and licensing assessments.

## 2.3 TEMPO

One of the areas of research is in surveillance and control systems in operation and maintenance. Here methods relating to sensor validation, condition monitoring and early fault detection are investigated. One of these methods is the use of data-reconciliation techniques. These techniques have been incorporated into a software package called TEMPO.

TEMPO: The thermal performance monitoring and optimisation system is designed to support plant personnel in identification and correction of problems which cause small decreases in plant efficiency over long periods of time. Parameter determination is performed by the following methods:

- Uncorrelated measurement uncertainties.
- Process flow sheet builder
- Non-linear flow sheet solver and optimizer
    - Reconciled measurement values
    - Parameter determination
- Linearization of flow sheet constraints
- Linear data-reconciliation for
    - Reconciled measurement uncertainties
    - Parameter uncertainty determination

## 2.4 VDI-2048

An important reference and comparative method is described in VDI-2048. This is a standard issued by 'Association of German Engineers' on 'Uncertainties of measurement during acceptance tests on energy-conversion and power plants'.

The method described here is build up of the following elements:

- Independent assessment of measurement uncertainties and correlations.
- Generation of data-reconciled measurement values using constraint equations.
- Parameter confidence limits calculation.

The assessment of measurement uncertainties is covered in chapter 4 of VDI-2048

### 2.4.1 Reconciled measurements

The reconciled solution is found by the successive linear data-reconciliation method. Where starting from an initial guess a linearized form of the process equations are determined. The optimal solution for this linearized problem is then determined. The resulting reconciled measurement values are then used as the starting point for a new linearization.

The values of the constraint equations aer then computed for the new starting point. This optimization method is then said to have converged when these new constraint equations are below a certain value as given in equation (143) in VDI-2048.

### 2.4.2 Reconciled uncertainties

As the system has been linearized the uncertainties in the actual measurement can be used to derive the uncertainties in the reconciled measurements. This step is normally performed after the optimal solution for the reconciled measurements has been found.

However, typically the reconciled uncertainties differ little with measurement values. This is because they are independent of how well the model fits to the measurements. From the optimised solution the reconciled uncertainties are calculated according to equation (124) in VDI-2048.

### 2.4.3 Parameter confidence limits

Non measured parameters that are derived from the reconciled measurements can also be calculated. As the full uncertainty matrix is known, variances and co-variances, then the uncertainty in these parameters can also be determined. It is important to include both variances and co-variances in this uncertainty determination as it is possible that it effects the result in either a positive or negative way.

The equation used for determining the parameter uncertainty is given, from eq (162) in VDI-2048, by:

$$s_{\Delta G}^{2} = \left(\frac{\partial \Delta g(\underline{x})}{\partial \underline{x}}\right) \bullet S_X \bullet \left(\frac{\partial \Delta g(\underline{x})}{\partial \underline{x}}\right)^T$$

where $s_{\Delta G}$ is the parameter uncertainty, $\Delta g(\underline{x})$ is the deviation of the calculated value of the parameter for a given set of reconciled measurements, $\underline{x}$, and $S_X$ is the uncertainty matrix for these reconciled measurements.

The derived uncertainties can then be used to determine confidence limits. The confidence limit is dependent on the probability distribution of the measurement values. As for VDI-2048, and most other applications of DR, the measurements uncertainties are specified for a standard Gaussian distribution. Thus the distribution of the derived parameters is also Gaussian and confidence limits can be given, from eq (164), by:

$$\Delta g(\underline{x}) + 1.96 s_{\Delta G} \geq 0.0$$

where 1.96 is the factor required for the 95% confidence limit.

# 3 Implementation at OKG

## 3.1 PROBERA system

The PROBERA system has been developed in-house by OKG for the purpose of process surveillance. The basic elements are as follows:

- Pre-processing of measurements with physical redundancy
- Process flow sheet builder
- Non-linear flow sheet solver and optimizer
    - Reconciled measurement values
    - Parameter determination
- Linearization of reconciled measurements dependence on parameters
- Linear data-reconciliation for
    - Reconciled measurement uncertainties
    - Parameter uncertainty determination

### 3.2 Measurement uncertainties

Measured process properties used by the flow sheet solver are assumed to be uncorrelated. Therefore any known correlations must be treated in a pre-processing phase. This can only be achieved for physically redundant measurements. These are measurements of the same property at the same place and are usually those measurements which have correlated uncertainties.

### 3.3 Flow sheet solver

The process to be modelled is built up using a graphical flow sheet editor. Flow sheet modules are connected by flows. Modules function by calculating the output flow parameters based on input flow parameters and the modules own parameters.

For the PROBERA system there are several different classifications of measurements:

- x Measurements associated with input parameters.
- px Measurements used for initialization of input parameters.
- y Measurements associated with output parameters.
- py Measurements which are not used in the data-reconciliation formulas.

The solver in PROBERA works by fitting a process simulation to measurement data. The process simulation is fitted by means of adjustable module parameters as well as input parameters associated with 'x' type measurement values.

The fitting process uses the down hill simplex method which is adequately described in the literature [e.g. Numerical Recipes].

The calculated values which are compared to the measurements are always solutions of the physical system.

So model properties corresponding to the i$^{th}$ measurement, are given by:

$$y_i = f_i\left(\{.., \mu_j, ..\}\right)$$

where all the 'x' type and 'y' type measurements are include in the measurement variables $\left(\{..., y_i, ...\}\right)$. And $f_i$ are functions of the model parameters $\mu_j$. The model parameters $\mu_j$ include all the module parameters as well as parameters associated with the 'x' type measurements.

The solver has the simple task of just minimising the object function. In this case the object function is the usual $\chi^2$ function,

$$\chi^2 = \sum_i \frac{\left(y_i^+ - y_i\right)^2}{\sigma_i^2}$$

where $y_i^+$ is the measured value of variable i, and $\sigma_i$ is the uncertainty in this measurement.

The minimum of this function is then found by the simplex method. The model parameters $\mu_j$ are varied until a minimum of $\chi^2$ is found. The convergence criterion is based on how close the current guess for $\chi^2$ is to its minimum value.

So in PROBERA the final solution will be a solution of the physical system, but may deviate from the optimal solution due to the limit of convergence.

This convergence criterion appears to differ from that used in VDI-2048, where the solution is at a minimum of the object function, but will differ from a physical solution by the convergence limit.

As in VDI-2048 the system is then linearized before the determination of variances. The solution values are used as the point to linearize around thus the model equations become:

$$y_i = y_{0,i} + B_{ij}\left(\mu_j - \mu_{0,j}\right)$$

where $B_{ij}$ is the parameter dependent part of the model matrix given by:

$$B_{ij} = \frac{\partial}{\partial \mu_j} f_i\left(\{..,\mu_j,..\}\right)$$

and $y_{0,i}$ and $\mu_{0,j}$ correspond to the point around which the system has been linearized. As stated earlier the solution of the non-linear optimization routine is used as the linearization point.

For VDI-2048 the objective of the convergence criteria is to ensure that the linearization point is the same as the optimized point, to within a given limit. The convergence criteria specified in VDI-2048 states that the weighted sum square of the equation contradictions (constraints) is below a specified value. The weighting used is the expected uncertainty in these contradictions based on the measurement values and uncertainties.

So we try to re-write the PROBERA method to comply with the VDI-2048 formula. To do this we re-write the model equations in terms of the linearization point, i.e.

$$\hat{\mu}_j - \mu_{0,j} = g_j\left(\{\ldots, y_{0,i},\ldots\}\right)$$

where $g_j$ is a function of the linearization point and $\hat{\mu}_j$ is the parameter value for the solution of the linear set of equations. This can then be set as a criteria that

$$g_j\left(\{\ldots, y_{0,i},\ldots\}\right) = 0$$

Following the terminology used in VDI-2048 this is called the auxiliary conditions, and **g**(**x**) is the vector of contradictions. The convergence criteria for VDI-2048 (eq. (143)) can then be written as:

$$\frac{\xi}{r} = \frac{\left(\{..,\hat{\mu}_j - \mu_{0,j},..\}\right) \cdot S_{\mu}^{-1} \cdot \left(\{..,\hat{\mu}_j - \mu_{0,j},..\}\right)}{r}$$

where r is the number of parameters and $S_\mu$ is the parameter uncertainty. This can be rewritten in terms of the measurement variables as,

$$\frac{\xi}{r} = \frac{1}{r} \sum_i \frac{\left(\hat{y}_i - y_{0,i}\right)^2}{\sigma_i}$$

where again $\hat{y}_i$ are the solutions of the linear data-reconciliation.

This last equation can be written using the same nomenclature as for the Adlers report as either,

$$\frac{\xi}{r} = \frac{1}{r} \hat{X}_P^T \cdot \left(M^T V^{-1} M\right) \cdot \hat{X}_P$$

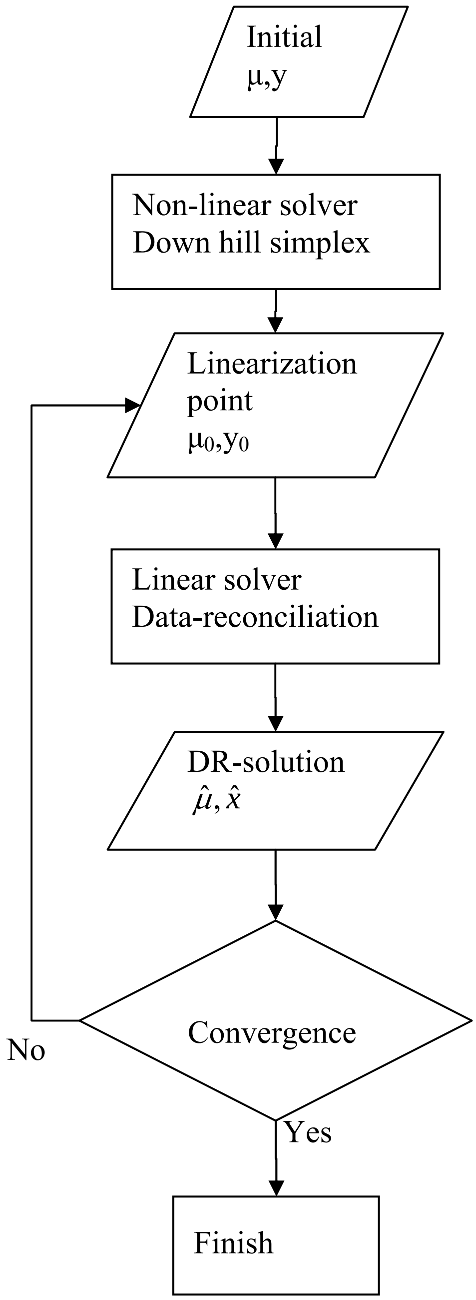
or

$$\frac{\xi}{r} = \frac{1}{r} \hat{Y}_P^T V^{-1} \hat{Y}_p$$

where

$$\hat{Y}_P = M \hat{X}_P$$

We then suggest the following calculation flow




Here we have drawn convergence loop to re-calculate the linear approximation for each iteration. So the simplex method is just used to find an initial starting point for this successive linear data reconciliation. Other calculation paths could be considered. The convergence criteria should  be specified regardless of the optimization technique as which technique gives the lowest computing time could be problem dependent.

## 3.4 Statistical determination.

### 3.4.1 Using Students distribution

The original calculations in PROBERA used a method based on the actual distribution of data. A term called the residual deviation (equation 2.13 Adlers) is calculated:

$$s_r^2 = \frac{\chi^2_{\min}}{r}$$

where r is the number of degrees of freedom.

This factor is then used to scale the uncertainty matrix for the parameters, (eq. 2.15 adlers)

$$C = s_r^2 \left(M^T V^{-1} M\right)^{-1}$$

As this is scaled due to the actual measurement variation then the confidence interval also needs to be scaled:

$$t_{r,z}\sqrt{C_{ii}}$$

where $t_{r,z}$ is the value of the Student distribution with r degrees of freedom at a confidence level of $\varepsilon$ where

$$z = 1 - \varepsilon/100$$

The authors of this report found this method to be inconsistent with other methods using data-reconciliation (VDI-2048, Madron). The main objection arises from the scaling of the parameter uncertainty matrix. This differs from the general principal of independent determination of measurement uncertainties.

The goodness of fit is normally used as a criterion to accept or reject the calculations.

Consider the case where there are 2 measurements of temperature that are physically redundant. The $\chi^2_{\min}$ value is effected by how close the measurements agree with each other. And from equation (2.15) in Adlers then the parameter uncertainty determination for ALL parameters is also effected. I.e. depending on the difference between the two temperature measurements the uncertainty determination of the model flow value is varied, and thus the determination of the thermal power is also varied. This is independent of the significance of these temperatures on the thermal power determination.

It soon becomes clear that any parameter determination will be subject to the model configuration, i.e. whether measurement averages are used or not.

### 3.4.2 Using Data reconciliation

The generally described method for data reconciliation makes use of analytical redundancy to improve state estimation. This is the application of process knowledge. For this method it is necessary to have a pre-determination of the measurement uncertainties. In the case of PROBERA the process information is contained in the model matrix, which relates the parameters to the measured variables:

$$Y_P = MX_p$$

where M is the model matrix (see definition of M, equation (2.4) Adlers).

We follow the same nomenclature as in the Adlers report. However we make some important distinctions for some specific values of the variables. These are

$X_P$ variable for model parameters, and ‘x’ type measurements describing all possible values for the parameter.

$\hat{X}_P$ The value of the model parameter which is a solution of the minimization.

$Y_P$ Variable for the model measurement values, ‘y’ and ‘x’ type measurements. This represents all possible values of the measurement outputs of the model.

$Y_P^+$ The $Y_P$ variable values when the actual measurement values are used.

$\hat{Y}_P$ The model output measurement values corresponding to the minimized $\hat{X}_P$ values.

Then the minimum solution of the $\chi^2_{\min}$ given this constraint equation is given by:

$$\hat{X}_P = \left(M^T V^{-1} M\right)^{-1} M^T V^{-1} Y_P^+$$

Here, the measurement corrections, $Y_P^+$ are the difference between measured values and converged solution (linearization point).The relationship between the measurement uncertainties and the model parameters can now be determined, and is given by:

$$S_{\hat{X}_P} = \left(M^T V^{-1} M\right)^{-1} M^T V^{-1} V \left(\left(M^T V^{-1} M\right)^{-1} M^T V^{-1}\right)^T$$

where $S_{\hat{X}_P}$ is the uncertainty matrix for the parameters and V is the uncertainty matrix for $Y_P^+$. After some matrix transformation and using the property that V is symmetric in its transverse then this can be reduced to:

$$S_{\hat{X}_P} = \left(M^T V^{-1} M\right)^{-1}$$

As the uncertainty matrix V has been pre-determined based on a normal Gaussian distribution then the same is true for the uncertainty matrix $S_{\hat{X}_P}$ . The confidence limits can then be determined based on this normal Gaussian distribution, e.g.

$$\hat{X}_{P,i} - u_z \sqrt{S_{\hat{X},ii}} \le \hat{X}_{P,i} \le \hat{X}_{P,i} + u_z \sqrt{S_{\hat{X},ii}}$$

where $u_z$ relates the standard deviation to the specified confidence limit, z, (i.e. for 95% confidence z= 0.05 and $u_z$ = 1.96).

In the implementation in PROBERA the thermal power is specified as a measurement of type 'y'. This is because with the particular configuration used the thermal power is an output of the simulation solver. The measurement associated with this thermal power is given a very large uncertainty. This will mean that this value has no influence on the final result and no influence on the reconciled uncertainty. This reconciled uncertainty is then the uncertainty in the calculated thermal power.

Given the definition of the model matrix and the parameter uncertainty matrix the uncertainty in the measurements is given by:

$$S_{\hat{Y}_P} = M\left(M^T V^{-1} M\right)^{-1} M^T$$

This matrix includes both the variance and the covariances. From this definition it can easily be seen that the reconciled measurements can also be defined as:

$$\hat{Y}_P = S_{\hat{Y}_P} \cdot V^{-1} \cdot Y_P^+$$

which for the reconciled value of the thermal power measurement is given by:

$$\hat{Y}_{P,th} = \sum_i \frac{S_{\hat{Y},th,i}}{\sigma_i^2} Y_{P,i}^+$$

Scaling both the measurements and the thermal power to their uncertainties gives the formula:

$$\left(\frac{\hat{Y}_{P,th}}{\sigma_{th}}\right) = \sum_i \left(\frac{S_{\hat{Y},th,i}}{\sigma_i \, \sigma_{th}}\right)\left(\frac{Y_{P,i}^+}{\sigma_i}\right)$$

It can also be shown that

$$\sum_j \left(\frac{S_{\hat{Y},th,j}}{\sigma_j \sigma_{th}}\right)^2 = 1.0$$

Thus we define a quantity called the relative contribution of each measurement uncertainty to the thermal power uncertainty as:

$$\left(\frac{S_{\hat{Y},th,j}}{\sigma_j \sigma_{th}}\right)$$

Other important values are the correlation matrix, adjustability, and the global and measurement acceptance tests.

**Correlation matrix**

The reconciled measurement correlation coefficient is defined as:

$$Correll_{ij} = \frac{S_{\hat{Y},ij}}{\sqrt{S_{\hat{Y},ii}} \cdot \sqrt{S_{\hat{Y},jj}}}$$

This relates how each measurement effects reconciled values of the other measurements. In this matrix only the off-diagonal terms are of interest, as the diagonal terms are all unity. Correlations close to 1, or -1, indicate a close link between the two measurements. So if errors are detected in one measurement (see measurement test below) then the other highly correlated measurements should also be investigated.

In VDI-2048, equation (129), this correlation coefficient is determined for the vector of corrections. This could easily be calculated from the PROBERA results. However, in the PROBERA implementation, the correlated uncertainties between the measurements are eliminated. As the only measurements with correlated uncertainties are those that are physically redundant, measuring the same quantity at the same location, the measurement uncertainties can and are adjusted to correctly compensate for this elimination. This simplifies the programming implementation whilst the resulting reconciled measurements and uncertainties are totally unaffected. Because of this elimination the measurement uncertainty matrix and the uncertainty matrix for the measurement corrections are not correct; only their sum, which is the uncertainty matrix for the reconciled measurements, is correct.

This correlation coefficient of the reconciled measurements is as good an indication of the interdependence of measurement results, as that proposed by VDI-2048. Therefore we do not see any reason not to use it instead. In fact for the simple case of averaging of measurements this correlation coefficient gives a stronger indication of interdependence than that for the measurement improvements. For averaging of two measurements the correlation coefficients are unity for both methods. However, as the number of measurements to average increases so the correlation coefficients for the measurement improvements decrease; whereas it remains as unity for the correlation coefficient of the reconciled measurements.

### Adjustability

The adjustability as defined by Madron is:

$$a = 1.0 - \frac{\sigma_{\hat{x}}}{\sigma_{x^+}}$$

This has a value between 0 and , where 0 indicated a non-redundant measurement and 1 indicates a fully redundant measurement. It is easily seen that for an adjustability of 1 the uncertainty in the reconciled measurement must be zero and thus the process state is fixed in the model.

The adjustably also gives an indication of how much the measurement value can be expected to be improved (adjusted) due to the other measurements and the physical model. This expected size of the improvement is also used as a statistical check in the measurement test.

### Global test

The objective of the solver is to minimise the object function, $\chi^2_{\min}$. The global test can then be used to determine if this minimum value is statistically likely. This test is given in VDI-2048 by equation (126), with expansion of $\xi_0$ by equation (107) as:

$$\xi_0 = \left(\hat{Y}_P - Y_P^+\right)V^{-1}\left(\hat{Y}_P - Y_P^+\right) \approx Y_P^+ V^{-1} Y_P^+ \leq \chi^2_{r,95\%}$$

Here the approximation can be used as the convergence criteria ensures that $\hat{Y}_P$ is small. It could also be argued that the $Y_P^+$ values should be used as they are actual solutions to the physical system. The value $\chi^2_{r,95\%}$ is the 95% confidence limit for the object function given *r* degrees of freedom.

Solutions exceeding this confidence limit should be rejected as there is a significant likelihood that a gross error is present in the measurements or that there is a significant modelling error.

### Measurement test

This test, as described by Narasimhan & Jordache, is also given in VDI-2048 as equation (128). This is written in the form used in PROBERA as:

$$r_{MT\,i} = \frac{\hat{Y}_{P\,i} - Y^+_{P\,i}}{\sqrt{V_{ii} - S_{\hat{Y}_P\,ii}}} \approx \frac{-Y^+_{P\,i}}{\sqrt{V_{ii} - S_{\hat{Y}_P\,ii}}}$$

where *i* and *ii* denote the individual measurements and the corresponding diagonal element of the uncertainty matrices. This equation is simply the measurement correction divided by its uncertainty. Again one can use the approximation that $\hat{Y}_P$ should be zero (to the order of the convergence). This approximation can cause problems when the uncertainty of the corrections is small, in the case of non-redundant measurements. In VDI-2048, equation (141), this problem is circumvented by placing a minimum value on the uncertainty of the measurement improvement. However, this minimum value is set arbitrarily and so not seen as significant improvement in the method.

VDI-2048 also places a limitation on the maximum value of the modulus of 1.96. This is the 95% confidence limit for the individual measurement. However, unlike the global test a value exceeding this limit does not necessary mean that there is a gross error. In fact as the number of measurements increase (actually increase in degrees of freedom) so does the chance that one of them by chance will have a correction greater than the 95% limit. A more comprehensive description can be found in Narasimhan & Jordache.

In our experience direct application of this test is not so useful in error detection. However, small changes in its value for any particular measurement can indicate faults. This is because much of the measurement uncertainty is associated with static parts of the instrument chain; so their contribution to the measurement correction is static from one time to another. It is therefore strongly recommended that this test value is trended for a time period (e.g. reactor cycle) and inspected for abnormal behaviour.

# 4 Comparison tests

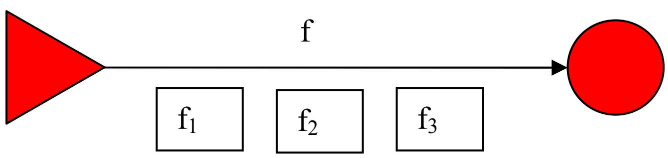
A data reconciliation method should not distinguish between physical and analytically redundant measurements. So the well known formulas for average and sum should also hold. These simple cases are also easy to verify, and so they have been used as the basis for some tests cases.

For each test case a PROBERA model was created and executed. The results were then compared to the expected values and any deviations noted.

## 4.1 Average

### 4.1.1 Average: Problem description

With several measurements associated with the same line, the line value will be interpreted as an average.



### 4.1.2 Average: Analytical solution

Here three flow measurements are associated with a single flow line. So the model matrix as defined in PROBERA looks like:

$$\begin{pmatrix} y_1 - f \\ y_2 - f \\ y_3 - f \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} (p_1 - f)$$

where f is the flow solution, $p_1$, a parameter representing the flow and $y_i$ the variables representing the measurement values. Then from equation (2.8) in Adlers the flow can be determined by:

$$(\tilde{p}_1 - f) = \left( \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2} \right)^{-1} \begin{pmatrix} \frac{1}{\sigma_1^2} & \frac{1}{\sigma_2^2} & \frac{1}{\sigma_3^2} \end{pmatrix} \begin{pmatrix} f_1 - f \\ f_2 - f \\ f_3 - f \end{pmatrix}$$

where $f_i$ are the actual measurement values.

So the reconciled flow will be the average of the three flows, or:

$$\tilde{p}_1 = \left( \frac{f_1}{\sigma_1^2} + \frac{f_2}{\sigma_2^2} + \frac{f_3}{\sigma_3^2} \right) \Big/ \left( \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2} \right)$$

And the uncertainty of this average value is given by:

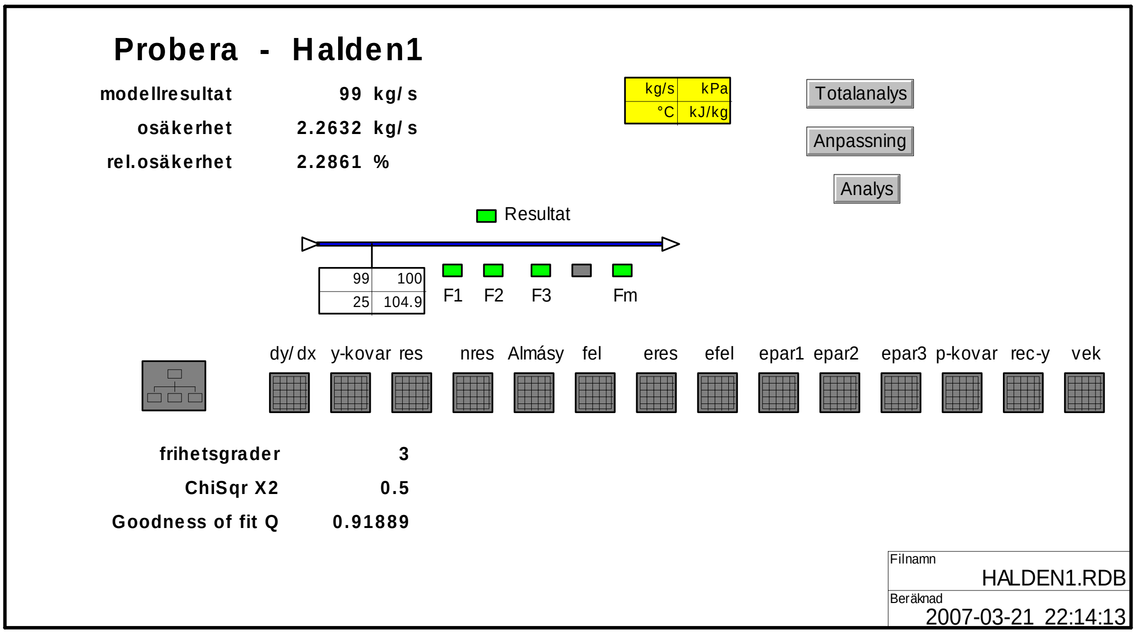
$$\sigma_{P1} = \left( \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2} \right)^{-1/2}$$

### 4.1.3 Average: Test data

From these equations a set of test data is generated for different cases. This test data is then used in PROBERA to confirm that the calculations are correct.

| Case | $f_1$ | $\sigma_1$ | $f_2$ | $\sigma_2$ | $f_3$ | $\sigma_3$ | f | $\sigma$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 1 | 100 | 1 | 100 | 1 | 100 | 0.57735 |
| 2 | 100 | 1 | 100 | 1 | 100 | 2 | 100 | 0.666667 |
| 3 | 100 | 1 | 100 | 2 | 100 | 2 | 100 | 0.816497 |
| 4 | 100 | 1 | 98 | 1 | 102 | 1 | 100 | 0.57735 |
| 5 | 100 | 1 | 98 | 1 | 99 | 1 | 99 | 0.57735 |
| 6 | 100 | 1 | 98 | 2 | 99 | 1 | 99.33333 | 0.666667 |
| 7 | 100 | 1 | 98 | 2 | 99 | 4 | 99.57143 | 0.872872 |

### 4.1.4 Average: PROBERA model



### 4.1.5 Average: PROBERA results

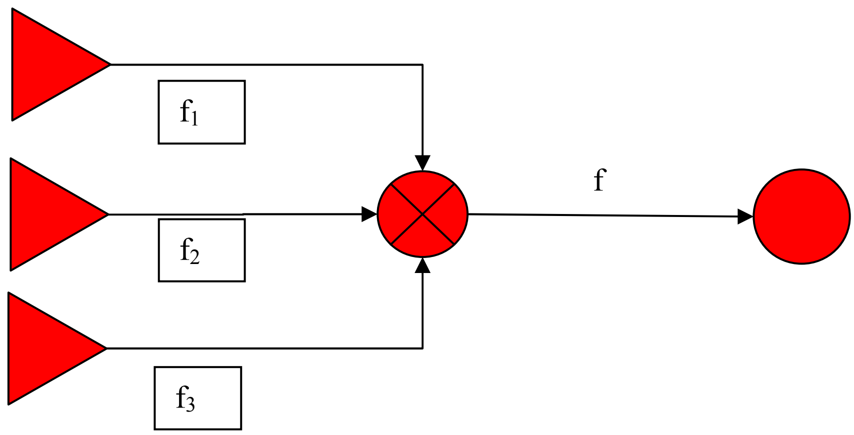
| | Test data | | PROBERA resutls | | Comments |
|---|---|---|---|---|---|
| Case | f | σ | f | σ | |
| 1 | 100 | 0.57735 | 100 | 0.57735 | Error message from Estim 2 |
| 2 | 100 | 0.666667 | 100 | 0.66667 | “ “ |
| 3 | 100 | 0.816497 | 100 | 0.8165 | “ “ |
| 4 | 100 | 0.57735 | 100 | 0.57735 | |
| 5 | 99 | 0.57735 | 99 | 0.57735 | |
| 6 | 99.33333 | 0.666667 | 99.333 | 0.66667 | |
| 7 | 99.57143 | 0.872872 | 99.571 | 0.87287 | |

Macro “Analys” ran until Par-estimering 2. This step was then removed.

## 4.2 Summation

### 4.2.1 Summation: Problem description

When parallel flows are mixed the resultant flow is their summation.



### 4.2.2 Summation: Analytical Solution

The flow from the mixer is the summation of the input flows. Here the summed flow and its uncertainty is given by:

$$f = f_1 + f_2 + f_3$$

And the uncertainty in f is given by:

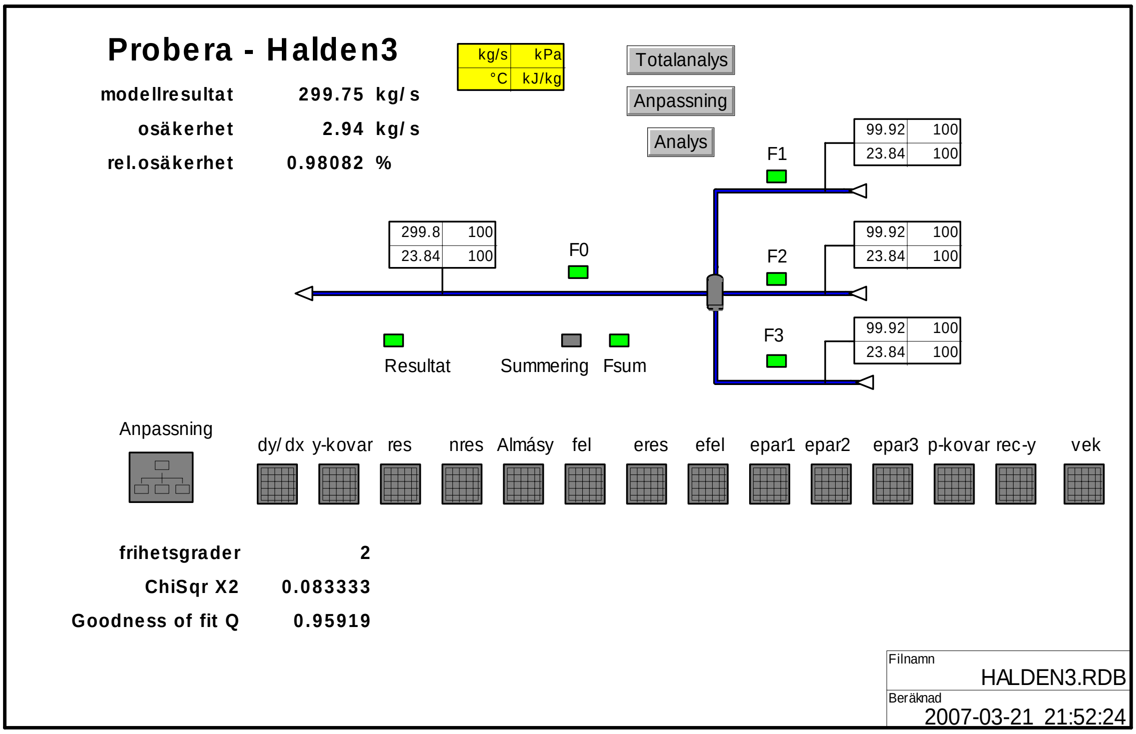
$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}$$

### 4.2.3 Summation: Test data

From these equations a set of test data is generated for different cases. This test data is then used in PROBERA to confirm that the calculations are correct.

| Case | $f_1$ | $\sigma_1$ | $f_2$ | $\sigma_2$ | $f_3$ | $\sigma_3$ | f | σ |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 1 | 100 | 1 | 100 | 1 | 300 | 1.732051 |
| 2 | 100 | 1 | 100 | 1 | 100 | 2 | 300 | 2.44949 |
| 3 | 100 | 1 | 100 | 2 | 100 | 2 | 300 | 3 |
| 4 | 100 | 1 | 98 | 1 | 102 | 1 | 300 | 1.732051 |
| 5 | 100 | 1 | 98 | 1 | 99 | 1 | 297 | 1.732051 |
| 6 | 100 | 1 | 98 | 2 | 99 | 1 | 297 | 2.44949 |
| 7 | 100 | 1 | 98 | 2 | 99 | 4 | 297 | 4.582576 |

### 4.2.4 Summation: PROBERA model



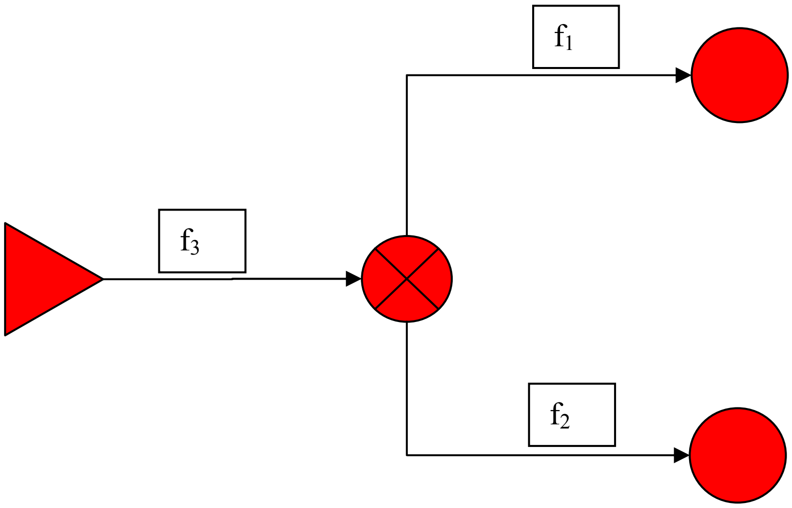

### 4.2.5 Summation: PROBERA results

| | Test data | | PROBERA results | | Comments |
|---|---|---|---|---|---|
| Case | f | σ (95%) | f | σ (95%) | |
| 1 | 300 | 3.39501 | 300 | 3.395 | Required to convert uncertainties to 95% confidence limit |
| 2 | 300 | 4.801269 | 300 | 4.8013 | |
| 3 | 300 | 5.88033 | 300 | 5.8803 | |
| 4 | 300 | 3.39501 | 300 | 3.395 | |
| 5 | 297 | 3.39501 | 297 | 3.395 | |
| 6 | 297 | 4.801269 | 297 | 4.8013 | |
| 7 | 297 | 8.982352 | 297 | 8.9823 | |

## 4.3 Flow splitting

### 4.3.1 Flow splitting: Problem description

The next test is a simple mixing flow.




### 4.3.2 Flow splitting: Analytical solution

Here there is a constraint that the sum of the reconciled flows $f_1$ and $f_2$ is the same as the $f_3$. They are then linked by the equation,

$$f_1 + f_2 = f_3$$

This can be written in matrix form as

$$a \quad + \quad Ax \quad = \quad 0$$

where

$$x = (f_1 \quad f_2 \quad f_3)$$

$$A = (1 \quad 1 \quad -1)$$

$$a = (0)$$

and the uncertainty matrix is

$$S = \begin{pmatrix} \sigma_1^2 & & \\ & \sigma_2^2 & \\ & & \sigma_3^2 \end{pmatrix}$$

Then the reconciled flows are given by

$$\tilde{x} = x^{+} + v$$

where v is the residual and is given by

$$v = -\frac{\left(f_1^{+} + f_2^{+} - f_3^{+}\right)}{\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right)}\begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \\ -\sigma_3^2 \end{pmatrix}$$

and the residual uncertainty matrix is given by

$$S_v = \frac{1}{\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right)}\begin{pmatrix} \sigma_1^2\sigma_1^2 & \sigma_1^2\sigma_2^2 & -\sigma_1^2\sigma_3^2 \\ \sigma_2^2\sigma_1^2 & \sigma_2^2\sigma_2^2 & -\sigma_2^2\sigma_3^2 \\ -\sigma_3^2\sigma_1^2 & -\sigma_3^2\sigma_2^2 & \sigma_1^2\sigma_1^2 \end{pmatrix}$$

The reconciled data uncertainties are then given by

$$S_{\tilde{x}} = S_x - S_v$$

### 4.3.3 Flow splitting: Test data

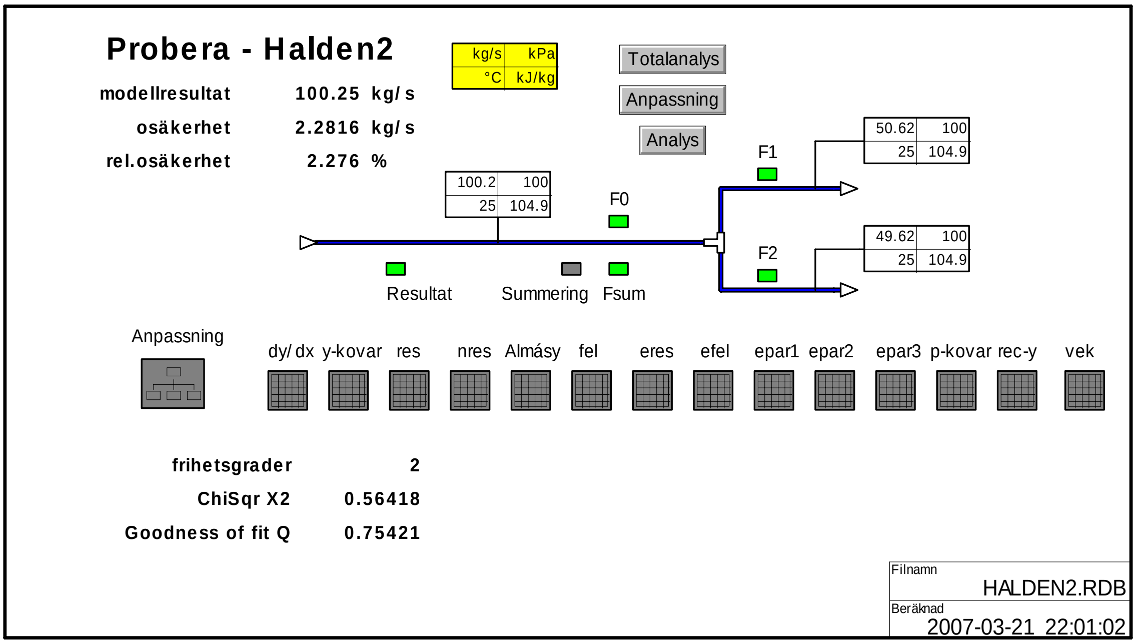
For test data

| Case | $f_1$ | $\sigma_1$ | $f_2$ | $\sigma_2$ | $f_3$ | $\sigma_3$ |
|---|---|---|---|---|---|---|
| 1 | 100 | 1 | 100 | 1 | 200 | 1 |
| 2 | 100 | 1 | 100 | 1 | 200 | 2 |
| 3 | 100 | 1 | 100 | 2 | 200 | 2 |
| 4 | 100 | 1 | 98 | 1 | 204 | 1 |
| 5 | 102 | 1 | 102 | 1 | 200 | 1 |
| 6 | 100 | 1 | 102 | 2 | 198 | 1 |
| 7 | 100 | 1 | 104 | 2 | 202 | 4 |

The results are

| Case | $f_1$ | $\sigma_1$ | $f_2$ | $\sigma_2$ | $f_3$ | $\sigma_3$ |
|---|---|---|---|---|---|---|
| 1 | 100 | 0.816497 | 100 | 0.816497 | 200 | 0.816497 |
| 2 | 100 | 0.912871 | 100 | 0.912871 | 200 | 1.154701 |
| 3 | 100 | 0.942809 | 100 | 1.490712 | 200 | 1.490712 |
| 4 | 102 | 0.816497 | 100 | 0.816497 | 202 | 0.816497 |
| 5 | 100.6667 | 0.816497 | 100.6667 | 0.816497 | 201.3333 | 0.816497 |
| 6 | 99.33333 | 0.912871 | 99.33333 | 1.154701 | 198.6667 | 0.912871 |
| 7 | 99.90476 | 0.9759 | 103.619 | 1.799471 | 203.5238 | 1.9518 |

### 4.3.4 Flow splitting: PROBERA model



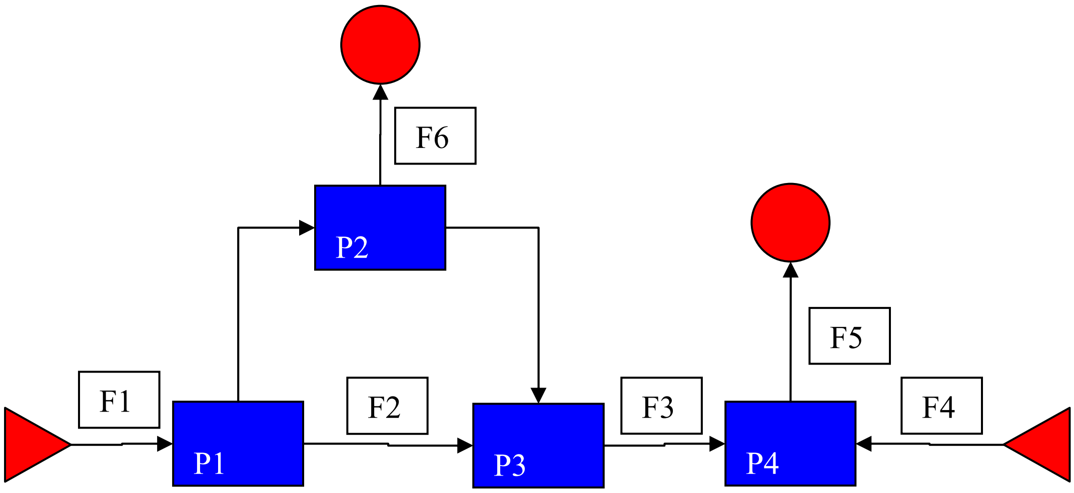

### 4.3.5 Flow splitting: PROBERA results

| Case | $f_1$ | $\sigma_1$ (95%) | $f_2$ | $\sigma_2$ (95%) | $f_3$ | $\sigma_3$ (95%) | Comments |
|---|---|---|---|---|---|---|---|
| 1 | 100 | 1.6009 | 100 | 1.6009 | 200 | 1.6009 | |
| 2 | 100 | 1.7893 | 100 | 1.7893 | 200 | 2.2633 | |
| 3 | 100 | 1.848 | 100 | 2.922 | 200 | 2.922 | |
| 4 | 102 | 1.6004 | 100 | 1.6009 | 202 | 1.6004 | |
| 5 | 100.67 | 1.6004 | 100.67 | 1.6009 | 201.33 | 1.6009 | |
| 6 | 99.333 | 1.7893 | 99.333 | 2.2633 | 198.67 | 1.7893 | |
| 7 | 99.9048 | 1.9129 | 103.619 | 3.5272 | 203.524 | 3.8257 | |

### 4.4 Text book example

#### 4.4.1 Text book: Problem description

This example is taken from MADRON, Example 4.8



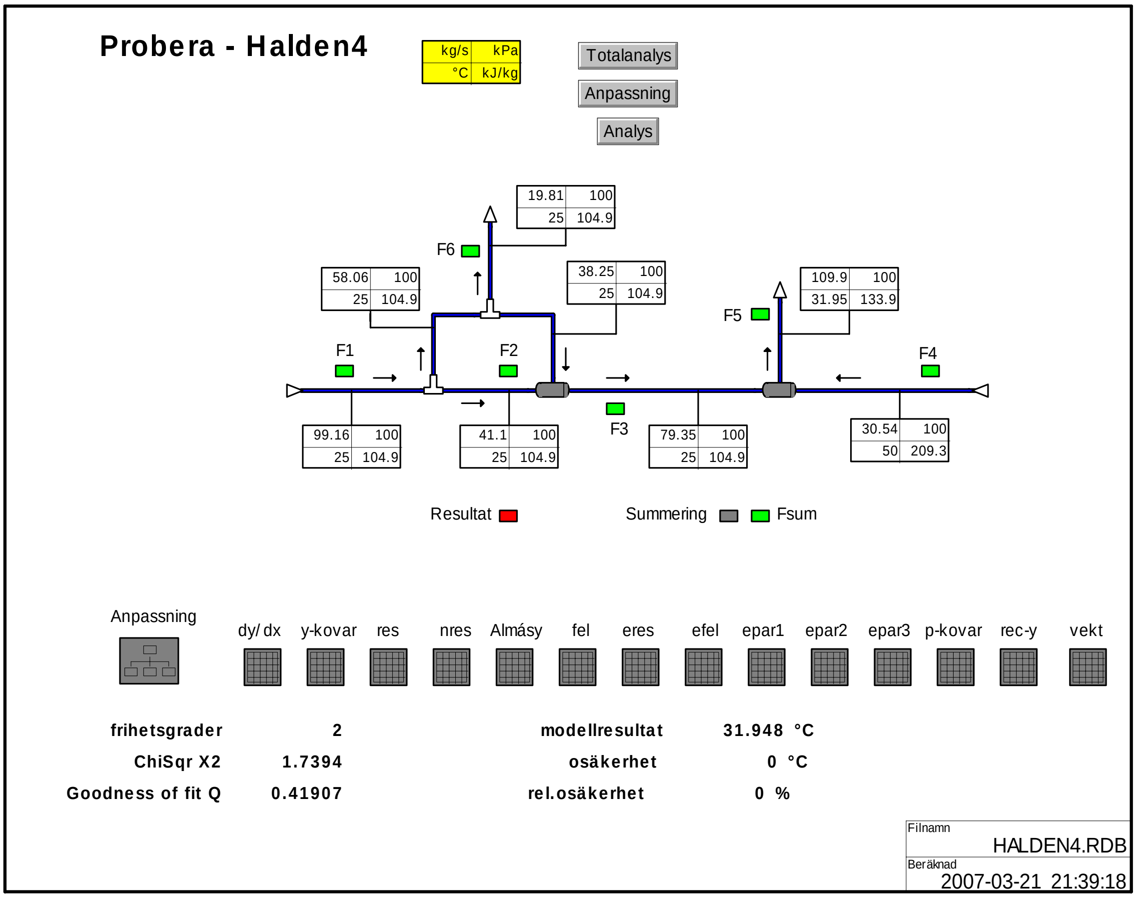

#### 4.4.2 Text book: Results

Here all but two flows are measured. The input and results for this test are:

| Flow Tag | $x^+$ | $\sigma_{x^+}$ | $v$ | $\tilde{x}$ | $\sigma_{\tilde{x}}$ | adjust |
|---|---|---|---|---|---|---|
| F1 | 100.1 | 1.0 | -0.942 | 99.2 | 0.60 | 0.40 |
| F2 | 41.1 | 0.8 | 0.000 | 41.1 | 0.80 | 0.00 |
| F3 | 79.0 | 0.8 | 0.349 | 79.4 | 0.60 | 0.25 |
| F4 | 30.6 | 0.4 | -0.063 | 30.5 | 0.39 | 0.02 |
| F5 | 108.3 | 2.0 | 1.590 | 109.9 | 0.69 | 0.65 |
| F6 | 19.8 | 0.1 | 0.009 | 19.8 | 0.10 | 0.00 |

IFE 

### 4.4.3 Text book: PROBERA model




### 4.4.4 Text book: PROBERA results

| Tag | $x^+$ | $\sigma_{x^+}$ | $v$ (1) | $\tilde{x}$ | $\sigma_{\tilde{x}}$ | adjust | Comment |
|---|---|---|---|---|---|---|---|
| F1 | 100.1 | 1.0 | -0.942 | 99.2 | 0.60 | 0.40 | |
| F2 | 41.1 | 0.8 | 0.000 | 41.1 | 0.80 | 0.00 | |
| F3 | 79.0 | 0.8 | 0.349 | 79.3 | 0.60 | 0.25 | (2) |
| F4 | 30.6 | 0.4 | -0.063 | 30.5 | 0.39 | 0.02 | |
| F5 | 108.3 | 2.0 | 1.590 | 109.9 | 0.70 | 0.65 | (3) |
| F6 | 19.8 | 0.1 | 0.009 | 19.8 | 0.10 | 0.00 | |

(1) Measurement residual is not directly calculated by PROBERA but has been determined from difference between reconciled and actual measurements.

(2) Difference in reconciled measurements. Rounding error present in text book example.

(3) Difference in reconciled measurement uncertainties. Rounding error present in text book example.

# 5 Implementation of PROBERA calculations.

The PROBERA calculations for the O1, O2 and O3 reactors are performed in separate models. This is necessary as there are significant process differences between them. These models were assessed in the following way.

First a visual inspection of the flow sheet was made. We (IFE) did not make a direct comparison to actual plant process sheets, but just considered that the flow sheet represented a physical process. This means that the components had a physical nature and were reasonable for a primary circuit. We also considered the positioning of the measurements used.

Next we considered the number of degrees of freedom. It is important to understand the origins of these degrees of freedom as they represent where modelling information has been used to correct process measurements. It is also important to determine if there are any unobservable parameters in the model. These are parameters whose values cannot be uniquely determined from the measurement information.

In all models there were sufficient measurements to describe the process. The degrees of freedom were mostly coming from physically redundant measurements. Here the data reconciliation technique will automatically fit the model state to the average of the physically redundant measurements. The remaining degrees of freedom are discussed for the individual models below.

In all the models the radiation loss of the whole circuit was concentrated in a single component. This localized the heat loss and represents a possible modelling error. However, the influence of this component is very small, and it's influence on the reactor thermal power value and uncertainty is most likely independent on its placement. In all models there were no indication of any modelling errors when considering the fitting.

Finally, the relative contributions to the thermal power uncertainty were inspected. The major contributory measurement uncertainties were identified. These were then compared to their expected contribution when considering the method described for CONDIN.

## 5.1 PROBERA, O1-Condin

This model [O1-PROB] makes use of 20 measurements and 13 parameters. Some of these measurements have physical redundancy. These are

411.K507 and 411.K508, the temperature before the feed water heater FV4

312.K501 and 312.K03, the temperature after the feed water heater FV4

211.KA113 and 211.KB112, the output steam pressure.

312.K309.PROB, 312.K310.PROB and 312.K311.PROB, the feed water flow.

As physical redundancy is also an analytical redundancy (as the model value is an average of the measurement values) then their correlation coefficients were also 1.0.

Also the input uncertainty for the thermal power is set to $10^{20}$ and so plays no practical role in the parameter uncertainty determination. This leaves us with 14 unique measurement positions and 13 parameters. Thus there is only 1 degree of freedom. From closer inspection of the flow sheet this arises between the two temperature measurements: 313.K5PROB and 321.K501, the temperature of HC flow and the temperature after the 321 pump. Although they are not physically redundant they are only separated by a pump with fixed properties. The temperature difference across the pump is dependent on the pressure head and the isentropic efficiency. As expected these two measurements also have a correlation coefficient of 1.0.

There are no other significant measurement correlations. This is as expected given the number of degrees of freedom.

From the relative contributions it can be seen that the uncertainty is dominated by the uncertainty in the flow measurement. There are three other measurements which have a significant influence on the thermal power uncertainty. These are: 312.K501, 312.K503 and the FUKTPROB measurement. This is consistent with the calculations used in the CONDIN code, where the thermal power is given by the formula:

$$Q_{tot} = Q_{turb} + Q_{crud} + Q_{cl} + Q_{rad} - Q_{pump}$$

The result of this equation is dominated by the $Q_{turb}$ term, which also dominates the uncertainty. The value $Q_{turb}$ is given by the feed water flow rate multiplied by the difference in enthalpy between the feed water and the output steam:

$$Q_{turb} = m_{mava} \cdot (h\mathring{a}(P, x_{\mathring{a}nga}) - hf(P, T_{mava}))$$

From the list of the relative contributions the feed water flow is seen as the largest contributor. It also only affects the value of $Q_{turb}$. This formula is then scaled to the total uncertainty in the thermal power and the uncertainty in the feed water flow.

$$\left(\frac{Q_{trub}}{\sigma_{th}}\right)=\left(\frac{m_{mava}}{\sigma_{m_{mava}}}\right)\cdot\left(\frac{(hå(P,x_{ånga})-hf(P,T_{mava}))}{\sigma_{th}}\cdot\sigma_{m_{mava}}\right)$$

The relative contribution from the feed water flow is then given by:

$$\left(\frac{(hå(P,x_{ånga})-hf(P,T_{mava}))}{\sigma_{th}}\cdot\sigma_{m_{mava}}\right)=\left(\frac{(2769-686.5)_{kJ/kg}}{11.46_{MW}}5.44_{kg/s}\right)=0.988$$

This is identical to the combined contribution from the 3 feed water flow measurements as seen in the table below (uncertainty contributions are combined as the square root of the sum of their squares). Here the uncertainties used are the 1 sigma values.

**Relative contributions to thermal power uncertainty, O1.**

| Measurement Tag | Contribution |
|---|---|
| 313.K3PROB | -0,0018 |
| 313.K2PROB | -0,0029 |
| 331.K324 | -0,0001 |
| 321.K504 | -0,0259 |
| 321.K501 | 0,0017 |
| 441.K507 | -0,0003 |
| 441.K508 | -0,0003 |
| 312.K501 | -0,0798 |
| 312.K503 | -0,0884 |
| 313.K5PROB | 0,0076 |
| 354.K301 | 0,0028 |
| 354.K576 | -0,0053 |
| 260.K090 | 0,0000 |
| LOSSPROB | 0,0262 |
| 211.KA113 | -0,0060 |
| 211.KB112 | -0,0060 |
| 312.K309.PROB | 0,5706 |
| 312.K310.PROB | 0,5706 |
| 312.K311.PROB | 0,5706 |
| FUKTPROB | -0,0864 |

## 5.2 PROBERA, O2-Condin

This model makes use of 18 measurements and 12 parameters, and as for the O1 model there are pairs of physically redundant measurements. These are:

3 flow measurements for each of the feed water flow lines. This combined with the inclusion of the thermal power measurement with an uncertainty of $10^{20}$ gives the number of degrees of freedom to 1.

Again this degree of freedom is seen in the analytic correlation between measurements 313K5PROB and 321K501FILT. These are the temperature of the HC flow and the temperature after the 321 P1-P2 pump. These two flows both originate from the splitter t 321 in the model. The only components between this splitter and the temperature measurements are pumps. These pumps have only a introduce week variance in the temperature, thus the high degree of correlation between the temperature measurements.

In a similar way to O1 the calculation, the thermal power uncertainty is dominated by the feed water flow uncertainty, as seen in the table below. The next significant contributions come from the feed water temperature, followed by the moisture content of the steam. We see that the moisture content has a lesser effect on the thermal power uncertainty here than for the O1 model. This is because the uncertainty in the measurement value is lower in the O2 model than in the O1 model.

Again for the main contributing part we calculate independently the contribution to the uncertainty. Here as there are two separate feed water lines these are considered individually:

$$\left( \frac{(2772 - 780.4)_{kJ/kg}}{9.86_{MW}} 3.47_{kg/s} \right) = 0.700 = \sqrt{3 \times 0.4043^2}$$

and

$$\left( \frac{(2772 - 782.1)_{kJ/kg}}{9.86_{MW}} 3.47_{kg/s} \right) = 0.700 = \sqrt{3 \times 0.4039^2}$$

**Relative contribution to thermal power uncertainty, O2.**

| Measurement Tag | Contribution |
|---|---:|
| 313K2PROB | -0,00107 |
| 313K3PROB | -0,00255 |
| 313K5PROB | 0,002014 |
| 331K503FILT | -0,0115 |
| 312K509/1FILT | -0,0939 |
| 312K508/1FILT | -0,09483 |
| 321K501FILT | 0,000317 |
| 260 K090 | 9,86E-20 |
| 211K116FILT | -0,00286 |
| 312K310PROB | 0,404291 |
| 312K312PROB | 0,404291 |
| 312K301PROB | 0,404291 |
| 312K311PROB | 0,403953 |
| 312K313PROB | 0,403953 |
| 312K302PROB | 0,403953 |
| 331K3PROB | 0,006413 |
| LOSSPROB | 0,020251 |
| FUKTPROB | -0,04134 |

## 5.3 PROBERA, O3-Condin

This model makes use of 28 measurements and 15 parameters, and as for the O1 model there are pairs of physically redundant measurements. These are:

211KA201COND and 211KB201COND which are the HC pump pressure head.

211KA560COND, 211KB560COND, 211KC560COND and 211KD560COND which are the steam output temperature.

211KA101COND, 211KB101COND, 211KC101COND and 211KD101COND which are the reactor pressure.

312KA301PROB, 312KA302PROB and 312KA303PROB which are one line of the feed water flow.

312KC301PROB, 312KC302PROB and 312KC303PROB are the feed water flow for the other line.

After considering these physical redundancies and the inclusion of the thermal power measurement with an uncertainty of $10^{20}$ gives the number of degrees of freedom as 1.

Again this degree of freedom is seen in the analytic correlation between measurements 312KC502COND and 312KA502COND, which are the temperature measurements on the two feed water flow lines. These temperatures are modelled as identical as they have a common boundary condition which is the feed water input to the model.

From the flow sheet for O3 it can be seen that there is not the same analytical redundancy between temperature measurements as seen in O1 and O2.

Again the uncertainty in the thermal power is dominated by the feed water flow, as seen in the table below. The feed water temperature is also an important factor. The other significant contributions come from the 331 system flow and the output steam moisture content.

Again for the main contributing part we calculate independently the contribution to the uncertainty. Here, as there are two separate feed water lines, these are considered individually.

$$\left( \frac{(2771 - 928.7)_{kJ/kg}}{11.88_{MW}} 2.74_{kg/s} \right) = 0.425 = \sqrt{3 \times 0.2452^2}$$

and

$$\left( \frac{(2771 - 928.7)_{kJ/kg}}{11.88_{MW}} 5.54_{kg/s} \right) = 0.859 = \sqrt{3 \times 0.4958^2}$$

**Relative contribution to thermal power uncertainty, O3.**

| Measurement Tag | Contribution |
|---|---|
| 211KX3PROB | -0,0017 |
| 354 KB301 | 0,0066 |
| 331 KB502 | -0,0020 |
| 321KB506COND | -0,0062 |
| 312KC502COND | -0,1878 |
| 312KA502COND | -0,1878 |
| 211KA201COND | -0,0012 |
| 211KB201COND | -0,0012 |
| 211KA560COND | -0,0002 |
| 211KB560COND | -0,0002 |
| 211KC560COND | -0,0002 |
| 211KD560COND | -0,0002 |
| 321KB501COND | 0,0114 |
| 260 KW951 | 0,0000 |
| 211KA101COND | -0,0033 |
| 211KB101COND | -0,0033 |
| 211KC101COND | -0,0033 |
| 211KD101COND | -0,0033 |
| 312KA301PROB | 0,2452 |
| 312KA302PROB | 0,2452 |
| 312KA303PROB | 0,2452 |
| 312KC301PROB | 0,4958 |
| 312KC302PROB | 0,4958 |
| 312KC303PROB | 0,4958 |
| 321KB301PROB | 0,0750 |
| LOSSPROB | 0,0247 |
| FUKTPROB | -0,0678 |
| 331KB3PROB | -0,0266 |

# 6 Conclusions

The method of uncertainty determination as implemented in PROBERA has been compared to the standard as specified in VDI-2048. Although not identical to the standard the differences were considered to have no effect on the resultant uncertainty.

The differences occur in the mathematical description of the problem. In PROBERA a flow sheet method is used to determine the model equations. This necessitates the introduction of model parameters, variables not directly associated with measured values. The minimization solution is thus required to solve for these model parameters simultaneously as for the measurement variables. As the correct construction of the model equations is essential for the data reconciliation method we consider the PROBERA approach to be the correct method to use. Note: the VDI-2048 standard gives no recommendations as how to construct the model equations.

After the implementation of the convergence criteria described in this report, the solution will be identical to that obtained without using model parameters. Thus, it is our opinion that the method of solution is of no importance; only the convergence criterion is of importance.

These differences are only in the intermediate stages of the mathematical treatment. The resultant reconciled uncertainties, and parameter uncertainties will be identical.

Several test cases were executed within PROBERA. These showed no deviations from the expected results.

Finally, the models for the O1, O2 and O3 reactors were considered. These were found to be correctly implemented. The cross correlations were investigated and found to be consistent with expectations. The relative contributions were also considered and the major contributory factors checked by direct calculation.

We therefore can say that the derived uncertainties of the thermal power are a reasonable representation of the true uncertainty.

## 7 References

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[O1-PROB] B. Persson, *Oskarshamn 1 - PROBERA-model för bestämning av osäkerheten i beräkningen av fissionseffekten med CONDIN*, OKG 2007-03015.

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[O3-PROB] B. Persson, *Oskarshamn 3 - PROBERA-model för bestämning av osäkerheten i beräkningen av fissionseffekten med CONDIN*, OKG 2007-03018.

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